ON REGENERATION AND COLOURATION IN ACTIVE ACOUSTICS SYSTEMS

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Abstract

In active acoustics, controlled feedback is often used to extend a room's reverberation time by regeneration. This brings an inherent risk of instability, while at stable gains a poorly conditioned feedback system can exhibit quality-limiting colouration. The gain before instability is usually estimated using frequency-domain eigenvalue measures computed from the open-loop response, and information about colouration or equalization is sometimes inferred from this. In this paper, we show that while eigenvalue analysis indicates the onset of instability, it does not reliably predict colouration while the system remains stable. We first demonstrate examples where the closed-loop behaviour for stable gains is not predictable by analysing the eigenvalues. We then demonstrate that singular value decomposition (SVD) of the closed-loop operator can be used as a loop-gain-dependent analysis tool. Analysis of simulations and measurements shows a clear correspondence between the characteristics of the SVD analysis and the emerging resonances in the closed-loop response. Furthermore, a spectral selectivity metric is proposed, which indicates the presence of colouration directly from the SVD analysis.

Keywords: *reverberation enhancement, feedback control, colouration*

1. Introduction

Active acoustics systems exploit controlled feedback loops to augment the reverberant response of a room. These systems typically comprise a sparse array of microphones and loudspeakers, and a digital signal processor (which we refer to as the "controller") that applies a matrix of impulse responses and stabilising equalisations between the microphones and loudspeakers [1]. By adjusting the *loop gain*, i.e., the electronic gain in the feedback path, an active acoustics system can extend the reverberation time of a venue, making it suitable for a broader range of musical performances. Such controlled regeneration of sound through the feedback loop forms the basis of regenerative reverberation enhancement, where energy is iteratively recirculated through the room to prolong the acoustic decay.

Although closed-loop stability is ultimately determined by pole locations, explicit pole modelling is generally impractical in multichannel acoustic systems, so stability is usually assessed from the open-loop transfer matrix instead [2]. The gain before instability (GBI) of multichannel acoustic feedback systems is usually determined by analysing the open-loop transfer matrix. The open-loop operator describes one circulation of energy around the system [3]. Eigenvalue decomposition (EVD) of the open-loop matrix provides a frequency-dependent set of complex eigenvalues. Instability occurs when any gain-scaled eigenvalue reaches the point $1 + j0$, i.e., corresponding to the Barkhausen criterion, which may be viewed as the eigenchannel form of the Nyquist stability criterion [2]. Due to the practical difficulties of reliably measuring the eigenvalue phase, the eigenvalue magnitudes are typically used to predict the threshold of instability [4] and for system equalization [5].

Operating an active acoustics system near the threshold of instability can induce audible colouration, manifested as spectral non-uniformity and resonant ringing in the closed-loop response [3], [6], [7]. Colouration arises when certain frequency bands decay more slowly than others, compromising the quality of the enhanced reverberation. To quantify colouration, researchers have proposed metrics based on the closed-loop impulse response from source to receiver [6], [8]. However, measuring colouration from the closed-loop response requires either full system implementation, or prediction of the closed-loop response based on open-loop simulations [9] or measurements [10]. As a result, designers often lack a practical means to predict colouration based only on the open-loop measurements available during the initial calibration phase.

One fundamental limitation of the EVD is that, under

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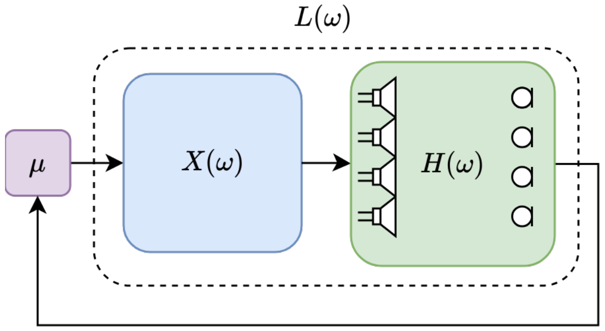

Figure 1: Block schematic of active acoustics system showing the loop gain μ, the electronic processor $X(\omega)$ and the acoustic system $H(\omega)$.

scalar loop gain, the eigenvalue-magnitude spectrum is invariant in shape and changes only by vertical scaling. It can therefore indicate which frequency will become unstable first as gain increases, but not how the stable closed-loop response evolves below the threshold. Singular value decomposition (SVD), by contrast, provides gain-dependent analysis of amplification and nearness to instability [11].

In this paper, we analyse acoustic feedback systems using the singular values of the closed-loop operator. The dominant singular value yields a gain-dependent regeneration function, from which we compute the local spectral deviation. This prediction indicates high gain modes directly from the open-loop transfer matrix, without requiring full closed-loop simulation or commissioning. Such high-gain modes typically lead to perceptual colouration in the closed-loop system.

The paper proceeds as follows. Section 2 provides mathematical background, comparing eigenvalue and singular value analyses and highlighting eigenvalue analysis limitations below the GBI. The SVD is then explored as an analysis tool for regeneration and colouration. Section 4 presents results computed from simulated and measured data, demonstrating how the metrics track closed-loop behaviour with increasing gain. Section 5 discusses design and equalisation implications of the findings.

2. Theory

A system diagram of a multichannel feedback system is shown in Figure 1. The open-loop transfer matrix $\mathbf{L}(\omega) = \mathbf{X}(\omega)\mathbf{H}(\omega)$ describes one circulation of energy through the room, microphones, controller and loudspeakers. The corresponding closed-loop operator

$$\mathbf{S}(\omega, \mu) = (\mathbf{I} - \mu\,\mathbf{L}(\omega))^{-1} \tag{1}$$

describes the overall closed-loop response after repeated recirculation at loop gain μ and angular frequency ω, where $\mathbf{I}$ is the identity matrix.

2.1. Eigenvalue analysis

Eigenvalue analysis is typically performed independently at each frequency bin of the measured open-loop transfer matrix. If $\mathbf{L}(\omega_k)$ denotes the complex microphone-to-microphone loop matrix at some discrete frequency bin ω_k, then it can be decomposed as

$$\mathbf{L}(\omega_k) = \mathbf{V}_k \boldsymbol{\Lambda}_k \mathbf{V}_k^{-1}, \tag{2}$$

where $\mathbf{V}_k$ is the matrix of eigenvectors and $\boldsymbol{\Lambda}_k = \mathrm{diag}(\lambda_{1,k}, \lambda_{2,k}, \ldots, \lambda_{I,k})$ is the matrix of eigenvalues with rank I. The eigenvalues $\lambda_{i,k}$ provide insight into the stability of the system with frequency. For scalar loop gain μ, the gain-scaled loop is

$$\mu\mathbf{L}(\omega_k) = \mathbf{V}_k\,(\mu\boldsymbol{\Lambda}_k)\,\mathbf{V}_k^{-1}, \tag{3}$$

so the eigenvectors are unchanged and every eigenvalue is scaled by μ. The open-loop dominant eigenvalue magnitude spectrum (*eigenspectrum*) therefore scales with loop gain, but the overall shape is invariant to the loop gain.

This method is effective because it is directly connected to the Barkhausen boundary condition; instability can only occur when some eigenvalue reaches the critical point $1 + 0j$, i.e. when $\mu\lambda_i(\mathbf{L})$ has unit magnitude and zero phase.This is equivalent to testing the loop gain at which $\mathbf{I} - \mu\mathbf{L}(\omega)$ becomes singular for each ω. For this reason, open-loop eigenvalue analysis provides a useful GBI estimation per frequency, and by extension we can set μ such that all frequencies remain stable.

However, even at stable gains, strong regeneration can cause energy to build up unevenly across frequencies and directions. In general, the behaviour described by the eigenvalues is not a precise measure of closed-loop pole proximity at a given gain, and even less so at any gain. Instead, the stable-side regeneration evolves non-uniformly with loop gain: resonances can appear, disappear, broaden, or shift as poles rotate relative to the Barkhausen condition. In this case, analysis that can predict the system behaviour at some gain below instability would be valuable.

2.2. Failure modes of eigenvalue analysis

Two failure modes illustrate how the open-loop EVD can mischaracterise regeneration and colouration below the instability boundary.

The first failure mode is non-normality. Omitting frequency-dependence for clarity, when $\mathbf{L} = \mathbf{V}\boldsymbol{\Lambda}\mathbf{V}^{-1}$ with ill-conditioned $\mathbf{V}$, the closed-loop operator satisfies

$$(\mathbf{I} - \mu\mathbf{L})^{-1} = \mathbf{V}\,(\mathbf{I} - \mu\boldsymbol{\Lambda})^{-1}\,\mathbf{V}^{-1}, \tag{4}$$

so amplification depends not only on the diagonal factors $(1 - \mu\lambda_i)^{-1}$ but also on the orientation and conditioning of the modal basis. In active acoustics, sparse layouts, asymmetric paths, unequal delays, frequency-shaped filters, and mixed direct and reverberant coupling can all produce loops whose left and right modal directions are poorly aligned. Large closed-loop amplification can then appear without corresponding peaks

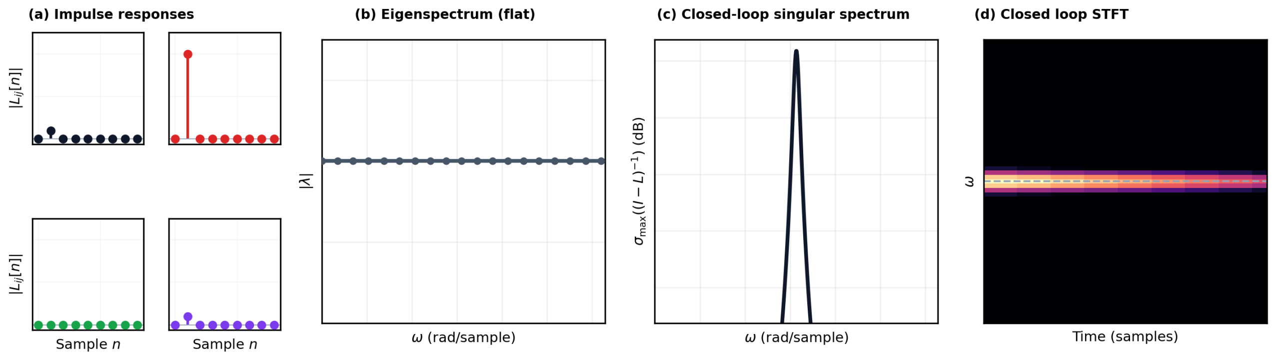

Figure 2: Non-normal 2x2 system. (a) impulse response of the open loop transfer matrix, (b) corresponding EVD magnitudes with frequency, (c) closed-loop singular spectrum with, (d) time-frequency response of the corresponding closed-loop response. Peak frequency corresponds to with the peak of (c).

in the eigenspectrum.
The non-normality condition is illustrated in Figure 2, which shows a highly non-normal open-loop leading to closed-loop resonance with no corresponding indicator in the eigenvalue magnitudes.

The second failure mode is gain-dependent pole rotation. A magnitude peak in the open-loop eigenspectrum indicates that, locally in frequency, a pole associated with that angle is likely to cross the instability boundary earlier than nearby poles. Using that same peak as a stable-side predictor of pole proximity, however, implicitly assumes that the relevant closed-loop poles move radially with loop gain, so that the same angle remains dominant on the approach to instability. Acoustic systems do not guarantee this. Delays, phase equalisation, crossover filters, modal overlap, and geometry-dependent propagation paths can all change the phase relationships inside the loop as gain increases, so closed-loop poles can rotate as well as move outward. Once that happens, an eigenspectrum peak that correctly predicts eventual unit circle crossing need not remain a valid predictor of pole proximity for a given subcritical gain. The dominant frequency can shift, peaks can appear or disappear, and the peak of the EVD magnitude trace need not line up with the peak of the regeneration envelope.
The pole rotation condition is illustrated in Figure 3. In this example, the dominant closed-loop poles move both radially and angularly as the loop gain is increased. At low gain, the strongest closed-loop response is concentrated around one frequency, but as the poles rotate toward the unit circle, the ringing shifts with them. The open-loop eigenspectrum still identifies the angle and μ at which the pole crossing eventually occurs, but fails to account for the trajectory.

2.3. Singular value analysis

Given the above discussion, it follows to examine whether any other, less constrained factorization can provide a more robust analysis than the open-loop EVD. To this end, we consider the SVD. For each frequency bin ω_k, the loop matrix may be written as

$$\mathbf{L}(\omega_k) = \mathbf{U}_k \boldsymbol{\Sigma}_k \mathbf{W}_k^{\mathrm{H}}, \tag{5}$$

where $\mathbf{U}_k$ and $\mathbf{W}_k$ are unitary matrices of left and right singular vectors, respectively, and

$$\begin{aligned} \boldsymbol{\Sigma}_k &= \operatorname{diag}(\sigma_{1,k}, \sigma_{2,k}, \ldots, \sigma_{I,k}), \\ \sigma_{1,k} &\geq \sigma_{2,k} \geq \cdots \geq 0. \end{aligned} \tag{6}$$

contains the singular values. Unlike the EVD, the SVD does not attempt to identify invariant modal directions of the loop. Instead, it identifies orthogonal input and output directions that are maximally coupled in gain. The singular values are therefore measures of directional amplification, rather than modal recurrence.

The improvement provided by the SVD depends on which operator is factorised. Applied to the open-loop matrix $\mathbf{L}(\omega_k)$, the dominant singular value $\sigma_{\max}(\mathbf{L})$ gives the largest one-pass directional gain through the loop, irrespective of whether that direction is an eigenvector. This partly addresses the non-normality problem, because it no longer relies on a well-conditioned eigenbasis. Applied instead to the return-difference matrix $\mathbf{I} - \mu\mathbf{L}$, the smallest singular value provides a direct measure of nearness to closed-loop singularity. It is zero if and only if the return difference is singular:

$$\sigma_{\min}(\mathbf{I} - \mu\mathbf{L}) = 0 \quad \iff \quad \det(\mathbf{I} - \mu\mathbf{L}) = 0.$$

For a normal loop matrix $\mathbf{L}$, the singular values of the return difference are the distances from the eigenvalues $\mu\lambda_i(\mathbf{L})$ to the critical point $1 + 0j$. The smallest singular value is the smallest of these eigenvalue distances:

$$\sigma_{\min}(\mathbf{I} - \mu\mathbf{L}) = \min_i |1 - \mu\lambda_i(\mathbf{L})|\,. \tag{7}$$

Thus, in the normal case, the eigenvalue-distance and singular-value descriptions coincide. In the non-normal case they separate: $\sigma_{\min}(\mathbf{I} - \mu\mathbf{L})$ can become small, revealing near-singularity, even when eigenvalue-

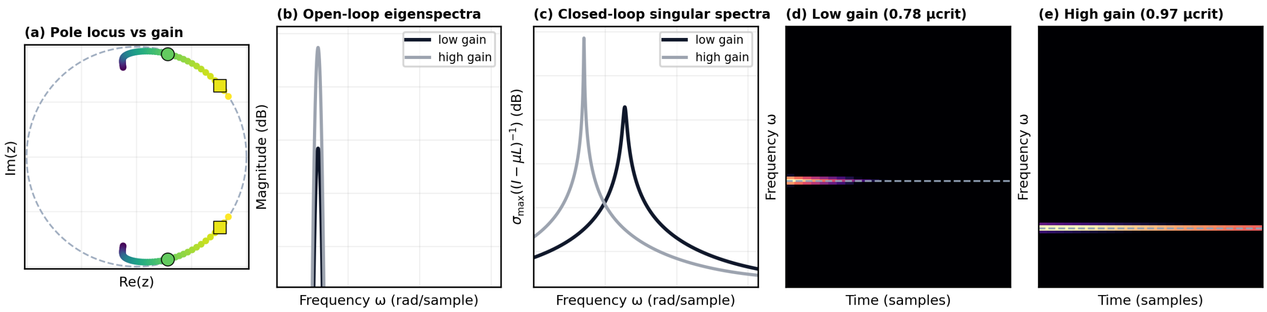

Figure 3: Pole rotation causing closed-loop ringing to drift with loop gain. (a) closed-loop poles spiral to intersection with the unit circle. Dot and square annotations show reciprocal pole positions for the low and high gain cases respectively. (b) Open-loop eigenspectra only indicate the ω and μ at which the poles cross, exhibiting the vertical shift with μ. (c) shows the closed loop singular value spectrum correctly characterising the closed-loop response. (d) and (e) closed loop response STFTs at two different loop gains, demonstrating moving instability.

magnitude plots remain comparatively flat.

The most direct analysis is obtained by taking the SVD of the closed-loop operator $\mathbf{S}$; its dominant singular value

$$e_{\text{svd}}(\omega, \mu) = \sigma_{\max}(\mathbf{S}(\omega, \mu)) \tag{8}$$

gives the worst-case closed-loop amplification, and therefore defines a gain-dependent regeneration function. Unlike open-loop decompositions, this factorises the convergent closed loop itself, after recurrence has been accumulated.

2.4. Estimating colouration with SVD

In the discussion above we have shown that applying SVD to $\mathbf{S}(\omega, \mu)$ yields $e_{\text{svd}}(\omega, \mu)$, a gain-dependent characterisation of the closed-loop system. We show in figures 2 and 3 that this characterisation does not suffer from the same failure modes as the open-loop eigenspectrum, defined as:

$$e_{\text{eig}}(\omega, \mu) = \max_i |\lambda_i(\mu\mathbf{L}(\omega))|. \tag{9}$$

Assuming that high gain corresponds to slow decay, colouration can be estimated from the local selectivity spectrum

$$\bar{e}(\omega, \mu) = \exp\left(\frac{1}{|\Omega_\omega|} \sum_{\nu \in \Omega_\omega} \log\big(\max(e(\nu, \mu), \varepsilon)\big)\right), \tag{10}$$

$$\xi(\omega, \mu) = \max\left\{20 \log_{10}\left(\frac{\max(e(\omega, \mu), \varepsilon)}{\max(\bar{e}(\omega, \mu), \varepsilon)}\right), 0\right\}, \tag{11}$$

where ε is a small regularization constant, ν is a frequency in the local neighbourhood, and

$$\Omega_\omega = \{\nu : |\log_2 \nu - \log_2 \omega| \leq 1/4\}$$

is a local log-frequency neighbourhood. The selectivity spectra ξ_{svd} and ξ_{eig} are computed by setting e as the corresponding amplification function e_{eig} or e_{svd}, respectively.

3. Experiments

Two experiments were designed to evaluate the stable-side behaviour of the proposed metric $\xi(\omega, \mu)$ using a simulated and measured acoustic system with a basic controller. The objective was to examine how the character of the closed-loop response evolves as the loop gain approaches instability.

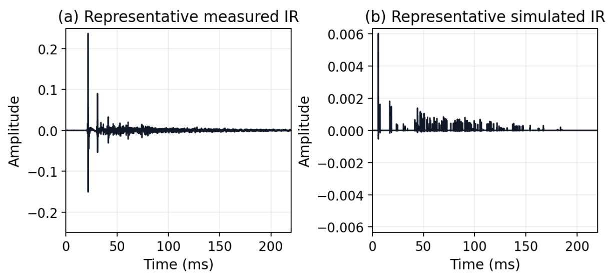

Figure 4: Time domain examples of measured (left) and simulated (right) RIRs.

3.1. Simulated room

The simulated experiment tested whether the theoretical failure of open-loop eigenspectra remains relevant in an acoustically plausible multichannel loop. A 4×4 loop was constructed from simulated room impulse responses (RIRs) and a structured two-tap controller. The RIRs were generated in a shoebox room (8 m $\times$ 12 m $\times$ 4 m) using fifth-order image sources in Pyroomacoustics [12]. Figure 4(a) shows an example RIR.

Figure 5 shows the closed- and open-loop analysis, with each row showing an increasing loop gain. The first column shows the Short-Time Fourier Transform (STFT) of the closed-loop impulse response, averaged across all microphones, representing the "ground-truth" acoustic properties of the loop. As the loop gain increases from top to bottom, the increase of regeneration and colouration is visible by the emergence of resonances which appear as vertical lines.

The second column shows the regenerative envelope

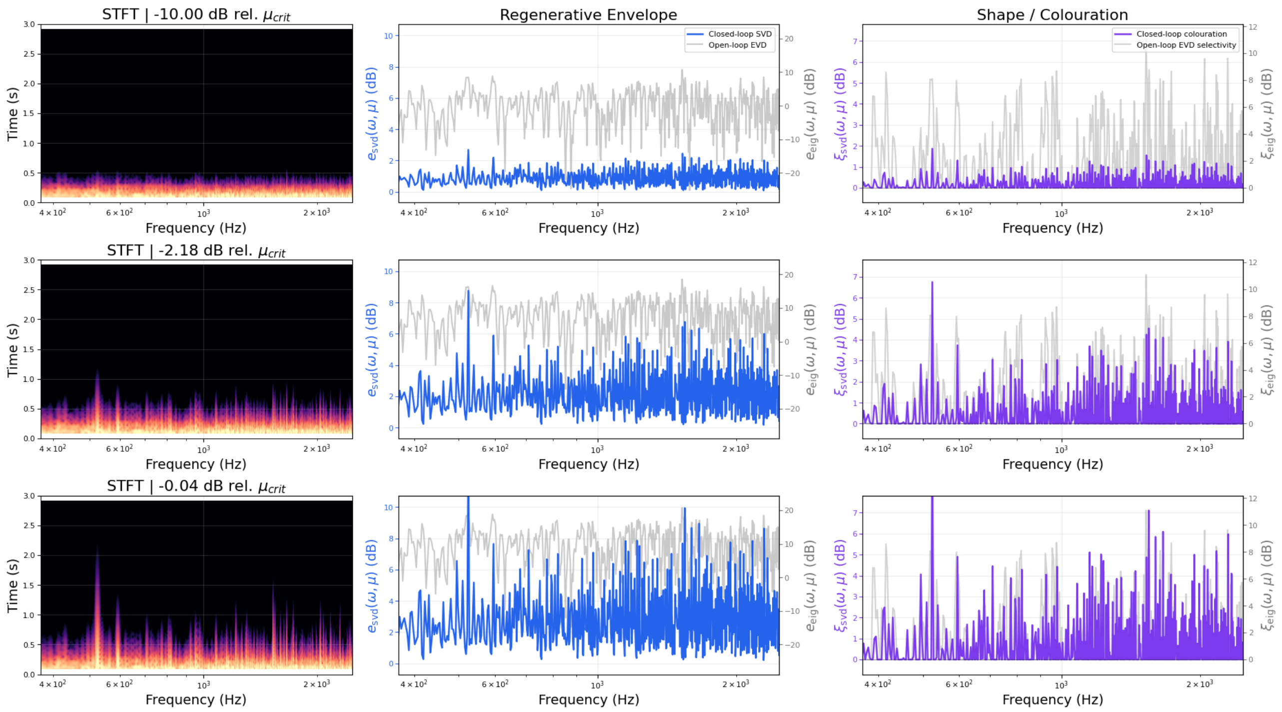

Figure 5: Closed-loop response and analysis of a simulated active acoustics system, with increasing a loop gain from top to bottom, showing the average microphone STFT in the closed loop (left), the computed regenerative envelope from the dominant singular spectrum of the closed-loop operator (centre), the corresponding eigenspectrum (centre, grey), and the extracted colouration spectrum from the regenerative envelope (right) vs the eigenspectrum (right, grey).

$e_{\mathrm{svd}}(\omega, \mu)$ predicted from the closed-loop SVD, alongside the maximum eigenvalue magnitude at each frequency $e_{\mathrm{eig}}(\omega, \mu)$. Whereas the EVD magnitudes exhibit a uniform upward shift with increasing μ, the closed-loop envelope reshapes with gain. Peaks sharpen and redistribute in frequency in a manner that mirrors the time-frequency response.

In the right column, the local spectral selectivity metric $\xi(\omega, \mu)$ is shown. The metric is computed from both $e_{\mathrm{svd}}(\omega, \mu)$ and $e_{\mathrm{eig}}(\omega, \mu)$, i.e., each curve attempts to quantify the spectral selectivity of the corresponding curves in the middle column. The SVD-derived spectral selectivity increases with loop gain, while the EVD-derived version has the same shape for every loop gain. It therefore cannot characterize the evolving colouration in the closed-loop response.

3.2. Measured data

Measured data was used to test whether the same effect survives in an experimentally measured loop, with real-world recordings in a 16×16 active acoustics system. Figure 4(b) shows an example of one of the RIRs. Figure 6 shows the predicted and measured time-frequency behaviour in the same format as Figure 5. It can again be observed that as the loop gain approaches the GBI, the increasingly strong ringing seen in the STFT is well-predicted by $e_{\mathrm{svd}}(\omega, \mu)$ and $\xi_{\mathrm{svd}}(\omega, \mu)$.

4. Discussion

It is useful to approach the system characterisation problem at three levels. Firstly, it is essential to predict at what gain and frequency the loop ceases to be stable. The EVD analysis of the open loop is a suitable tool for this purpose. Secondly, we can predict the regeneration strength, i.e., how strongly and uniformly the stable closed-loop amplifies and extends energy decay. Our results have shown that the SVD-derived measure $e_{\mathrm{svd}}(\omega, \mu)$ can be used for this, as worst-case loop amplification is likely to correspond to longer decay in the closed-loop. Finally, we can estimate the colouration from the open loop via $\xi_{\mathrm{svd}}(\omega, \mu)$. Unlike the existing approaches based on time-domain closed loop active RIRs, our measure avoids the need to infer colouration from secondary features, such as fitted energy-decay curves, STFT magnitudes, or energy decay reliefs.
There is an implicit objective in active acoustics calibration to maintain a spectrally even closed-loop response at a chosen operating gain. Our results imply that for this task, static filters derived from the open-loop eigenvalues may not be sufficient. Instead, whitening or stabilising equalisation must be conditioned on loop gain, because the relevant spectral peaks are properties of $(\mathbf{I} - \mu\mathbf{L})^{-1}$, not of $\mathbf{L}$ alone. In other words, the filter that best controls colouration at one gain need not be the filter that best controls it at another. Future work should further explore this aspect.

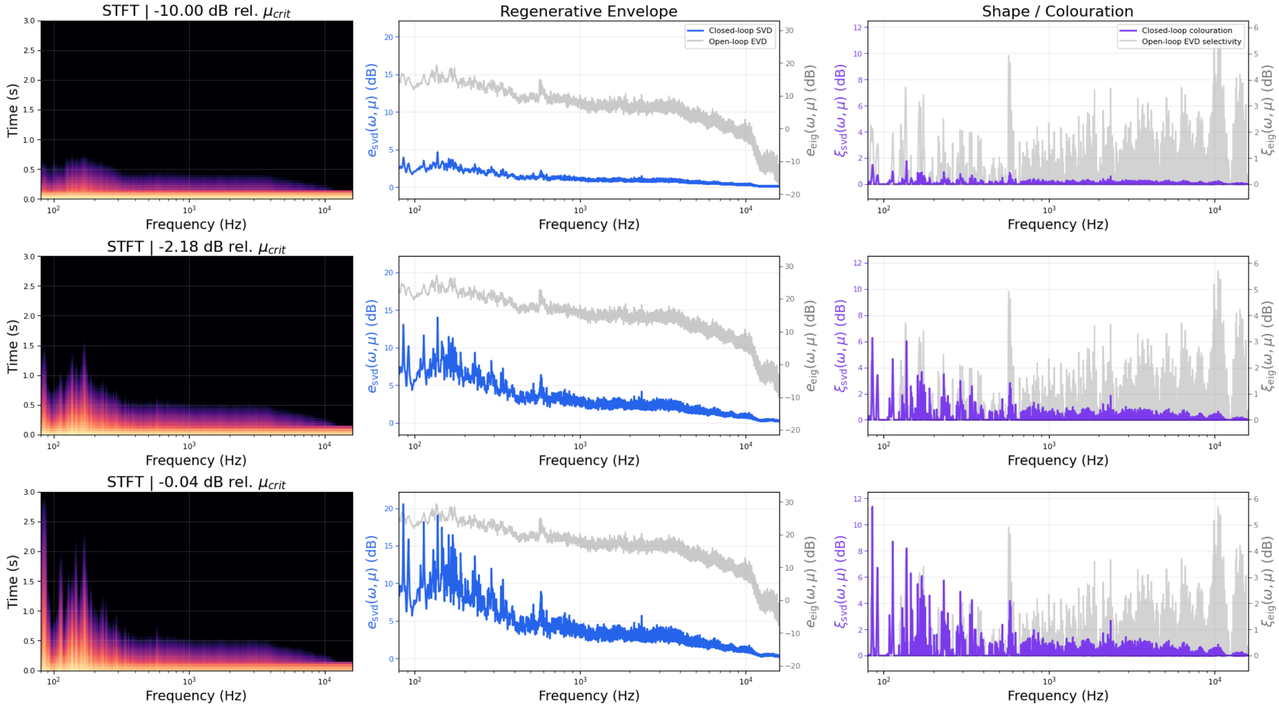

Figure 6: Closed-loop response (left) and analysis (centre, right) of the measured 16×16 active acoustics system with increasing loop gain (plots as described in Figure 5).

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